INFINITY AND DEITY

Glimpses of God in Mathematics

By John Noonan

Galileo is quoted as saying, "Mathematics is the alphabet with which God has written the universe."[1] The nineteenth-century mathematician Leopold Kronecker said, "God created the natural numbers; all else is the work of men." In a sense this is true; God revealed the natural numbers to us, and as we discover new mathematical truth, we move closer to understanding the universe. Indeed, though God knows all mathematical truth, he chose to reveal only the most basic truth to man.[2]

In certain academic circles, one speaks of integration of faith with one's discipline. This integration of faith refers to the idea that our faith informs our intellect and our intellect informs our faith. To better understand our faith, we look to our intellect and vice versa. Throughout the course of my career, I have found that the prevailing opinion among nonmathematicians is that there is little or no connection between mathematics and faith. This is simply not true.

We seek to know a God whose nature is beyond our experience. Just as the narrator in Edwin Abbott's *Flatland*[3] could not fully comprehend a third dimension (and the existence of simple geometric objects like the sphere), so we cannot expect to fully comprehend God, who transcends time and space.[4] Physically, God is now at the creation of the universe and at its end. I cannot comprehend a four-dimensional cube, particularly because I am bound to our three-dimensional world. I can, however, using mathematics, see a three-dimensional shadow of a four-dimensional cube.

1. This quote appears at the end of Disney's *Donald in Mathmagic Land* (1959). Though an excellent short film, it is not a scholarly source. On searching Galileo's writings, it appears that the Disney quote is a paraphrase of "Philosophy is written in that great book which continually lies open before us (I mean the Universe). But one cannot understand this book until one has learned to understand the language and to know the letters in which it is written. It is written in the language of mathematics, and the letters are triangles, circles and other geometric figures." Found in Galileo's *Il Saggitore* (1623). This is similar to the Disney version, but I prefer Disney's wording to Galileo's.
2. The issue of the existence of mathematical objects (whether we create them or discover them) will not be addressed here. Some of the references cited discuss issues of this nature.
3. Edwin Abbott, *Flatland: A Romance of Many Dimensions*. Originally published in 1884, recently published by Dover in 1992.
4. In John 8:58 and Colossians 1:15-17, the tense suggests that Jesus *is* at more than one point in history.

GALILEO IS QUOTED AS SAYING, "MATHEMATICS IS THE ALPHABET WITH WHICH GOD HAS WRITTEN THE UNIVERSE."

I believe the analogy is clear. We cannot see God, but we can see his shadow, his handiwork, and bits and pieces of his nature.

While it is valid to ask the question, "How does my faith inform my mathematics?" here I will focus on the question, "How does my mathematics inform my faith?" Deeper treatment of these ideas can be found in Nickel's *Mathematics: Is God Silent?*[5] and in a collection of papers entitled *A Christian Perspective on the Foundations of Mathematics*.[6]

Infinity and God

Though scripture abounds with references to God as an eternal being (infinite time), there are very few references in the Bible to other aspects of the infinite nature of God, most notably, Psalm 147:5; "Great is our Lord and mighty in power; his understanding has no limit"(NIV). My desire is not to argue God's infinite nature but to point out the implications of an infinite God.

Mathematics is the study of the infinite. We use calculus to measure with infinitely small increments. We work with infinite sets of numbers. We divide an interval into an infinite set of subintervals to determine the continuity of a function. I believe that much of what we learn about the infinite in mathematics is a reflection of God's character. The ancients had a problem with infinity. They saw paradoxes such as Zeno's paradox as proof that infinity did not exist. Now we understand these paradoxes more fully and yet the infinite still yields results that are counterintuitive. The same can be said of infinite God.

Infinity Rejected

Interestingly, the Greeks might not have accepted calculus, as they had a problem with the infinite. The Greek mathematician Zeno was famous for posing paradoxes that perplexed mathematicians of his day.[7] One of Zeno's paradoxes involves a runner who competes in a race. Before crossing the finish line, the runner would have to pass the halfway point. Before crossing that point, he would have to cross the one-fourth way point, before that, the one-eighth way point, and so on. If we allow for an infinite division of the racecourse, then the runner would have an infinite number of waypoints to cross before reaching the finish. Zeno thought that the runner could not possibly cross an infinite number of waypoints in a finite time. He had difficulty reconciling this with the fact that the runner could be observed to finish the race.

It was not until mathematicians understood infinity, or more specifically, calculus, that they were able to resolve Zeno's paradoxes: it is possible for the sum of an infinite set of numbers to be finite. Perhaps it is this cultural denial of infinity that caused the absence

5. James Nickel, *Mathematics: Is God Silent?* (Vallecito, CA: Ross House Books, 1990).

6. *A Christian Perspective on the Foundations of Mathematics* (Wheaton: Wheaton College, 1977).

7. Wesley C. Salmon, ed., *Zeno's Paradoxes* (Indianapolis: Hackett Publishing Co., 2001), 8–10.

of direct references to God's infinitude in our scriptures.

THE PREVAILING OPINION AMONG NON-MATHEMATICIANS IS THAT THERE IS LITTLE OR NO CONNECTION BETWEEN MATHEMATICS AND FAITH. THIS IS SIMPLY NOT TRUE.

Infinity Defined

Mathematicians didn't have a workable, rigorous definition of infinity until George Cantor's work in the late nineteenth century. To Cantor, a set of objects was considered infinite if it could be placed in one-to-one correspondence with one of its subsets. In other words, a set of objects is infinite if you can take some of the objects away and still have the same number. For example, one could count the number of grains of sand in a child's sandbox. The number would be very large, but certainly not infinite. If you were to take a bucket of that sand and dump it on the ground, you would have less sand in the sandbox. In contrast, if you consider the idea of eternity (an infinite number of days), today isn't any closer to the end of eternity than yesterday was. Whether we ascend to heaven immediately upon dying, or wait around in our graves until Christ returns does not affect the number of days we will spend in heaven.

To man, God is paradox. The very notion that Jesus could be fully human and yet fully God does not line up with our experience. Similarly, we have difficulty with the trinity; how can three be one? Certainly one of the first mathematical theorems we learn is that 1+1=2 (yes, mathematicians can prove this statement!) It is not until we admit the infinite that we encounter equations such as 1+1=1 (Jesus) and 1+1+1=1(the trinity).

Infinity and the Trinity

I remember as a boy pondering the nature of God. Specifically, I wondered how a single being could be comprised of three wholes. We are taught at a young age how to add, and I knew that 1+1+1 did not equal 1. Yet, with God it did. Then there is the problem of Christ: fully God and fully man. A complete acceptance of the trinity and the nature of Christ did not come until I studied infinite sets. I later discovered that other mathematicians had pondered these questions as well. Cassius Keyser[8] first presented the following explanation of the trinity: Cantor's definition of an infinite collection of objects states that a set is infinite if one of its subsets has the same number of things in it as the original set. For example, the set of whole numbers, {1,2,3,4,5 . . .} is infinite because it has the same number of numbers as one of its subsets; the set of positive even integers, {2,4,6,8,10 . . .}. The way that mathematicians check that two sets have the same number of things is by pairing them up. Most of us could check that we have the same number of fingers on our hands by holding them next to one another and pairing the fingers up. If every finger on your left hand has a corresponding finger on your right hand and vice versa then you have the same number on each hand. This pairing or

8. Cassius Keyser, *The Rational and the Superrational* (New York: Scripta Mathematica, 1952), 51–116.

WE CANNOT SEE GOD, BUT WE CAN SEE HIS SHADOW, HIS HANDIWORK, AND BITS AND PIECES OF HIS NATURE.

matching is usually done with a formula. The formula used to match the whole numbers to the positive even integers is the formula 2*n*. Thus, 1 is matched to 2, 2 is matched to 4, 3 is matched to 6 and so on. It is not hard to see that this formula will do the job of pairing these two sets. A similar formula could be found that matches the set {3,6,9,12,15 . . .}, the positive multiples of 3, with the whole numbers. Slightly more complicated formulas are needed for the sets {1,4,7,10,13 . . .} and {2,5,8,11,14 . . .} which represent the positive integers that are 1 more than multiples of 3 and 2 more than multiples of 3 respectively. Together, the three sets above make up the whole numbers, yet they do not overlap and all of them have the same number of entries as the whole numbers. Mathematicians use the Hebrew symbol $\aleph_0$ (pronounced by mathematicians as "aleph-naught") to describe the number of whole numbers. That symbol also describes the quantity of numbers in each of the three sets mentioned above. The inevitable conclusion is that $\aleph_0+\aleph_0+\aleph_0=\aleph_0$. If God is infinite, we shouldn't be surprised that he is made up of three distinct parts, each equal to the whole.

Paradox and Contradiction

In studying mathematics, I have noticed that whenever I come across a counterintuitive result, an apparent contradiction or a paradox, infinity lurks somewhere in the equation. One of the most famous paradoxes is Russell's Paradox,[9] named after the philosopher and mathematician, Bertrand Russell. Russell's paradox is a true one that exists because we accept that sets can be infinite. Without infinite sets, we would not have Russell's Paradox. Many of the mathematical objects I study and teach which include infinity are counterintuitive. Using an infinite interval, I can construct a surface with infinite surface area and finite volume.[10] Think of the implications of such a surface. It would be impossible to paint such a surface because its surface area is infinite, yet I could fill the surface with a finite amount of paint! The Koch Snowflake[11] is a geometric figure classified as a fractal that has finite area, but infinite perimeter. The outline of such a figure could not be drawn because it is infinitely long, yet it could be contained on a finite piece of paper.

Many of the doctrinal divisions that exist in the Church today might be reconciled if the parties involved could realize that God is infinite, and whenever we glimpse the infinite, our intuition is foiled. In fact, paradoxes abound. God is counterintuitive, seemingly contradictory, and even paradoxical at times.

In discussing doctrinal issues with other Christians, I find that certain topics are flashpoints. Examples include issues such as the eternal security of the believer, sovereignty vs. free will, and sanctification (entire vs. process). If I adopt an opposing viewpoint on such issues in a discussion, some Christians react as if I have questioned

9. A concise and understandable explanation of Russell's Paradox can be found in Keith Devlin's *Mathematics: The New Golden Age* (London: Penguin Books, 1988), 39.

10. The solid generated by rotating the function $y=1/x$ from $x=1$ to $x=\infty$ about the x-axis is one example.

11. Devlin, 78.

the inerrancy of the Bible. Reactions like "but the Bible clearly says that . . ." are common in such discussions.[12] The interesting thing is that those with the opposing viewpoint would say the same thing, citing references to support their position. As I see it, these contradictions exist in scripture and we have several choices to reconcile the differences. We could adopt one point of view and creatively interpret passages that seem to contradict our point of view. We could question the inerrancy of the Bible as it relates to passages that contradict our point of view. I find it preferable to accept the apparent contradictions as an attribute of our infinite God. Perhaps one day we will learn enough about the nature of God to reconcile the contradictions, but my view of the Bible and the God that breathed it are unshaken by these contradictions now. It's possible to believe in both eternal security and the notion that a believer can reject God's love and so fall away from his faith. Other seeming biblical inconsistencies can be reconciled with the understanding that God is infinite.

WHENEVER I COME ACROSS A COUNTER-INTUITIVE RESULT, AN APPARENT CONTRADICTION, OR PARADOX, INFINITY LURKS SOMEWHERE IN THE EQUATION.

Truth and the Infinite

Ever since Euclid produced his *Elements*, Mathematicians have sought truth through rigor and proof. For 2000 years, Euclidean geometry was accepted as the one true geometry that could be used to describe the spatial relationships within our universe. In the early 1800s, Carl Freidrich Gauss, Janos Bolyai, and Nicolai Lobachevsky independently began to study "new" geometries that were consistent within themselves, but were at odds with Euclidean geometry. The primary feature that distinguished these new geometries from Euclidean geometry were the assumptions they made about parallel lines. In Euclidean geometry, If you draw a straight line on a piece of paper and put a point somewhere on the paper but not on the line, it is assumed that there is only one line that can be drawn through the point that is parallel to the original line. This seems reasonable enough, but mathematicians were never able to prove that this is true. In fact, the emergence of new geometries in the early nineteenth century halted the search for a proof. The idea of parallel lines (two lines that never cross) can never be verified, for we can never extend two lines infinitely in both directions to check if they are parallel. We can only extend them a finite amount. The idea of lines being parallel requires one to admit the infinite.

The presence of so-called non-Euclidean geometries disturbed mathematicians enough to cause some to try to axiomatize arithmetic. That is, they wanted to list all of the assumptions that they needed in order to build proofs of all truth about arithmetic. If Euclidean geometry could be shown to have loopholes, what could be said of the mathematics that had developed since? Arithmetic was a logical place to start, for most of mathematics can be couched in terms of arithmetic and the counting numbers. If all of

12. The eternal security debate is one example. The eternal security camp quotes verses like John 10:27–28 while the opposing camp quotes verses like Matthew 24:13.

WHENEVER WE GLIMPSE THE INFINITE, OUR INTUITION IS FOILED.

the assumptions on which our arithmetic theorems were built could be listed, the same could be done for other branches of mathematics.

In 1900, mathematician David Hilbert gave an address at the International Mathematics Congress in Paris in which he outlined twenty-three problems which, if solved, would further mathematics more than any other results.[13] These problems became known as the Hilbert Problems. Most have been solved, but some are still unsolved. One of those listed was the problem of proving that the axioms of arithmetic are complete. In other words, Hilbert wanted a proof that all truth about arithmetic could be generated from a finite set of assumptions.

Like most mathematicians of his day, Hilbert was a modernist. He believed man could know all truth scientifically. Mathematics is the basis for modern science and so it was essential to him to resolve questions challenging the validity of mathematics. Of chief importance was the question of the consistency of our assumptions about arithmetic.

Bertrand Russell and Alfred Whitehead came up with a system of assumptions for arithmetic, but could not prove they were complete. They published their list in 1910 in their book *Principia Mathematica*.[14] During the course of their research, Russell discovered his famous paradox. This was another blow to the mathematical community. How could paradox exist in a system that is supposed to reveal truth?

Hilbert would live to see many of his problems solved. Kurt Gödel, a logician, solved the question of the completeness of the axioms of arithmetic in 1931.[15] The solution was not what anyone expected. Gödel proved that no finite set of assumptions would ever be sufficient to prove all truth about arithmetic. His proof showed that, given any finite set of assumptions, one could always produce a statement about arithmetic that could not be proved yet was true. Gödel further showed that if one were to add assumptions to the list that allowed the proof of the aforementioned statement, then one could produce a new statement that could not be proved with the new list of assumptions. The only way around this would be to have an infinite list of assumptions!

The shock waves from this discovery were felt far beyond the mathematical community. Indeed, our postmodern philosophy has elements of this result as its tenants. If mathematics couldn't prove all truth, then there had to be some truth that couldn't be known scientifically.

It has been over seventy years since Gödel proved that all truth couldn't be known scientifically, yet certain parts of the church still

13. David Hilbert, "*Vortag, gehalten auf dem internationalen Mathematiker-Kongress zu Paris 1900,*" *Nachriten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen* (1900): 253-297. English translation at http://aleph0.clarku.edu/~djoyce/hilbert/

14. Bertrand Russell, and Alfred North Whitehead, *Principia Mathematica*, 3 vols, (Cambridge 1910, 1912, 1913).

15. Kurt Gödel, "*Über formal unentscheidbare Sätze der Principia Mathematica und verwandter System, I,*" *Monatshefte für Mathematik und Physik* 38 (1931): 173–198.

cling to modernity. I would go so far as to point out that our holy scriptures are a finite description of an infinite God, and as such, there is truth that cannot be contained within their pages. I believe God revealed his essential truth through scriptures, but even Jesus, speaking to his disciples said, “When he, the Spirit of truth, comes, he will guide you into all truth” (John 16:13, NIV). Someday God may reveal truth through his Spirit that reconciles the contradictions that we struggle with and the paradoxes that perplex us. Until that time, we should remember that we have a finite description of an infinite God who knows no bounds and cannot be contained by human reason.

OUR HOLY SCRIPTURES ARE A FINITE DESCRIPTION OF AN INFINITE GOD, AND AS SUCH, THERE IS TRUTH THAT CANNOT BE CONTAINED WITHIN THEIR PAGES.

MARS HILL REVIEW

Number 20

Founder and Publisher
KIM HUTCHINS

Editor
SARAH KOOPS VANDERVEEN

Founding Editor
LIAM ATCHISON

Managing Editor
DAVID VANDERVEEN

Literary Editor
JILL LAMAR

Business Manager
MICHELLE HUTCHINS

Design and Layout
RACHEL WINSTEAD

Circulation Fulfillment
JANELLE JEFFERSON

Contributing Editors
SUSIE ALBERT MILLER & LOIS SWAGERTY

Mars Hill Review is published three times per year.

Subscription Rates: USA one year $36, two years $65; Canada one year $48, two years $89; for orders outside North America one year $63, two years $119. Make payments in U.S. funds drawn on U.S. banks. To order, please call 1-800-990-MARS, or visit www.marshillreview.com.

Unsolicited Submissions: Essays, scholarly studies, interviews, fiction, nonfiction, poetry, artist profiles, reviews, recommendations, and artwork are welcome. Submissions for longer articles should be between 2,000 and 5,000 words in length, written in a clear, journalistic style accessible to the general public. When sending a submission, include with your hard copy a diskette formatted in one of the following programs: Microsoft Word, WordPerfect, or ClarisWorks. Deadlines for issues are as follows: Winter/Spring, October 1; Summer, February 1; Fall, June 1. Response time to unsolicited manuscripts and queries is ten to twelve weeks. Enclose a self-addressed, stamped envelope. We are unable to return manuscript submissions. Send submissions to: *Mars Hill Review*, Editorial Department, P.O. Box 10506, Bainbridge Island, WA 98110-0506.

Visual Submissions: We are seeking to build a reference file of art to illustrate and complement our feature articles. Please feel free to submit pieces to us. Artwork cannot be returned unless the submission is accompanied by a self-addressed envelope or package with full postage cost enclosed.

Mars Hill Review is indexed in the *Religion Index of Periodicals, Religious and Theological Abstracts*.
Distributed by Ingram Periodicals, Inc., La Vergne, TN. Printed by Classic Printing, Nashville, TN.